Biovistas

International Journal of Biological Research

ISSN: 2321-0524
Volume: 10 **Issue:** 1

Article Type: Research Article

Differential Equations-Acoustic Wave Theory and Computational Logistic Model - Re-Analysis

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Abstract

To understand basics of differential equations and implementation in Acoustic Wave Theory, Runge Kutta method and its applications in real life. RANGE KUTTA METHOD [R.K METHOD] It is a weighted average of 4 slopes at each (tn, yn) The most used and classical one is presented here with average of 4 steps. Error (Error of Approximation) If y(x) is an exact solution and yx is an approximate solution obtained by one of the numerical methods then the difference between exact solution and approximate solution is called error. The step-size is changed as of required to get some predetermined accuracy in solution with minimum computational effort. By the results obtained we can come to a conclusion that in almost every considered differential equation the values of k2 and k3 is observed to be almost equal/same. On Comparison of Runge Kutta Method with Euler and Heun Method we can say that Runge Kutta Method is efficient and less time consuming while Euler and Heun Methods give the approximate value in 10 and 40 steps respectively.

Runge Kutta Method

- Parameters like maximum running times, step size etc. can be controlled.
- Working of numerical methods will be much clearer for students doing research on it.
- We compare the exact solutions and the numerical solutions, the results obtained by Runge Kutta Method fourth order are of order 10-20 for ϕ and 10-14 for θ.

Acoustic Wave Propagation

- Usually, the study is regarding the development of numerical solution techniques for simulating the interaction between acoustic and elastic waves.
- Much focused on mutual interaction between 2-time harmonic linear wave equations.
- The temporal discretization for time dependent equations was made by the second-order accurate central finite different scheme or the fourth order Runge Kutta approach.

Differential Equations

- We can solve these equations either by using analytical or numerical methods.
- Analytical solution exists rarely because it's quite complicated to solve

Keywords: Range Kutta Method, Acoustic Wave Theory, Derivative, Linear Functions, Calculus

Introduction

A kind of equation, to say a mathematical equation which gives a relation between a particular function and its derivatives. For Example: -Consider a quantity, to say physical quantity as a function and its derivatives says about rate of change of that particular quantity.

There are many perspectives for this depending on the solution identity like the easier way without full simplification, without finding the exact form.

Mathematical Representation

$$dy/dt+f(y)=0$$

Here f(y) is a function, and dy/dt is derivative of the function f(y) with respect to time and this forms a differential equation.

Function with Derivatives

Derivative is a way to represent rate of change that is the amount by which a function is changing at one given point. In general derivatives of linear functions are constant i.e., the derivative in one spot on the graph will remain the same on another.A function's derivative can be used to search for the maxima and minima of the function by searching for places where its slope is zero.

Received: 30.10.2021; **Revised:** 25.01.2021; **Accepted:** 01.02.2022

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Derivatives are Fundamental Tool of Calculus.

The symbols dx, dy, etc. were introduced by Gottfried Wilhelm Leibnitz in 1675. It says a functional relationship between dependent and independent variables. The symbols f',f'',f''… was introduced by Lagrange.

Fundamentals of Differential Equation

An equation containing the derivative of unknowns is called a differential equation. An equation involving in derivative of only one independent variable is called an ordinary differential equation.

Order of a Differential Equation

Order of differential equation is defined as the order of the highest derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

Degree of a Differential Equation

The power of highest order (if exists) present in the given differential equation is called the degree of differential equation.

Formation of Differential Equation

A differential equation can be obtained by differentiating the given equation 'n' number of times. Where 'n' is the number of constants in the equation. OR Eliminating the constants by differentiating the equation.

Methods of Solving Differential Equations

Variable Separable Form

Bring the equation with same variables on both the sides and then integrate to obtain the differential equation.

Homogenous Differential Equations

Let f (x, y) be a function containing 2 variables x and y, replace x and y by λx, λy respectively in the above function for any non-zero constant λ.

A function f (x, y) is said to be homogenous function of degree n if f (λx, λy) = λnf (x, y).

Rules to solve:

$$f(x,y) = \frac{dy}{dx}$$
$$substitute\ \frac{y}{x} = v$$
$$y = v * x$$
$$\frac{dy}{dx} = v + \frac{xdv}{dx}$$

and then integrate.

Linear Differential Equations

A differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

is called a linear differential equation.

Rules:

Write the given equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x only.

Find Integrating Factor which is given by

$$I.F = e^{\int pdx}$$

Solution is given by :

$$y * I.F = \int Q * (I.F)\, dx + C$$

Existence of Solutions

General Solution

Solving a differential equation and finding its solution will be the general solution of differential equation.

Particular Solution

Substituting the given conditions to the general solution of differential equation and obtaining a equation.

Range Kutta Method [R.K METHOD]

It is a weighted average of 4 slopes at each (t_n, y_n). The most used and classical one is presented here with average of 4 steps.

General Formula

$$S_1=f\ (t_n,\ yn)$$
$$S_2=f\ (t_n+\varepsilon/2,\ y_n+S_1/2 * \varepsilon)$$
$$S_3=f\ (t_n+\varepsilon/2,\ y_n+S_2/2 * \varepsilon)$$
$$S_4= f\ (t_n+\varepsilon,\ y_n+S_3 * \varepsilon)$$
$$y_{n+1}=y_n+ (S_1+ 2S_2+ 2S_3+ S_4) * \varepsilon/6$$

As of consideration and previous studies, using the value of ε=0.5, twice as large as for Euler and Heun methods[1, 2]. We need to remember that f (t, y) =4t√y.

While the above formula is applied, we get some results which is the preliminary step.

From those results we get y.

Using the obtained y, we get 2nd step results in the same format.

In these 2 steps we get a better approximation in R.K Method while the Heun and Euler methods give it in 4 and 10 steps respectively.

The local error in this method was found to be in the order of ε5.

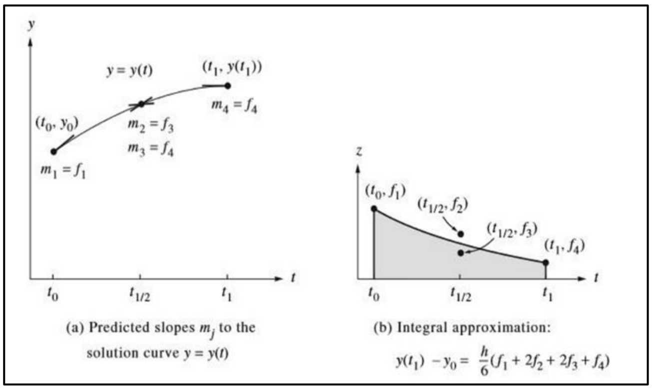
The global error was found to be in the order of ε4.

On plotting a graph for the results obtained it gives a graph.

On the exact solution curve and these lines segments doesn't match the exact solution curve at other places.

This small error can be rectified by reducing the step size or using a smoother interpolation curve.

Coming to analyze R.K Method, it’s actually a family of implicit and explicit iterative methods, this includes the well-known routine called Euler method.
These methods were developed by
C. RUNGE and M.W KUTTA around 1900.
We have the formula for Euler method as
yn+1=yn+hf (xn, yn)
which arrives from a solution xn→xn+1≡ xn+h
so, this formula is unsymmetrical.



Reasons to not use Euler’s Method

1) Not stable
2) The method is not very accurate when compared to other.

1st order R.K Method [Euler’s Method of Order 1]

y(x1) ≈y1=y0+k1 where k1=hf (x0, y0)

2nd order R.K Method [Modified Euler’s Method]

y(x1) ≈y1=y0+(k1+k2)/2 where
k1=hf (x0, y0) and k2= hf (x0+h, y0+k1)

Error (Error of Approximation)

If y(x) is an exact solution and yx is an approximate solution obtained by one of the numerical methods then the difference between exact solution and approximate solution is called error.

Error=Exact Solution-Approximate solution
=y(x)-yx
=y(x1)-y1 at x=4

R.K Method Order 1—Error in O(h2)
R.K Method Order 2—Error in O(h3)
R.K Method Order 3—Error in O(h4)
R.K Method Order 4—Error in O(h5)

Step Size Control For R.K Method

The step-size is changed as of required to get some predetermined accuracy in solution with minimum computational effort.

Implementation of adaptive step size control requires that the stepping algorithm signal information about its performance, most important, an estimate of its truncation error. when error is large, we reduce the step size to get some accurate value than the previous value. [8]

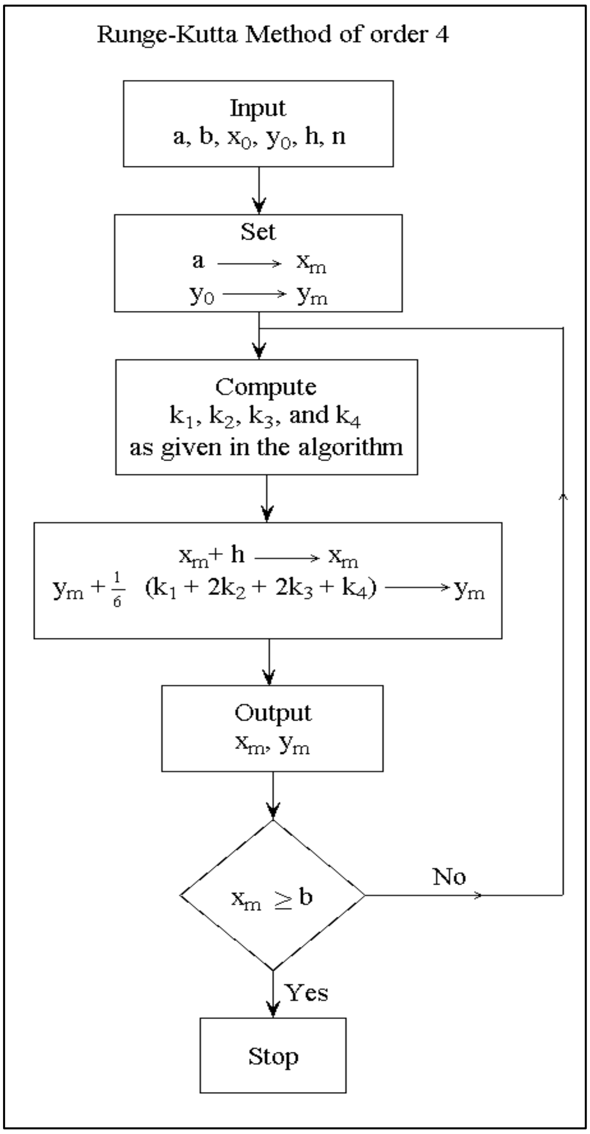
Numerical Results of the Two Methods

We study the Rossler choatic attractor with a=0.1 , b=0.1 and c=14.
The program above has been run separately for orders 4 and 5 with initial condition y0=(1 1 0) and step size 0,05 .The resulting errors and program runtime is given below. [6]

RK4 Method			RK5 Method		
Step Size	Error	Time	Step Size	Error	Time
0.025	35.7235940083	25.0	0.025	4.32464747225	10.6
0.0125	21.9453774473	44.9	0.0125	0.114162427377	21.3
0.00625	1.07721196989	108.7	0.00625	0.00373384751883	41.7
0.003125	0.0658815896626	115.8	0.003125	0.000118970270334	82.7
0.0015625	0.00418440282466	113.6	0.0015625	3.91436761227e-006	158.2
0.00078125	0.000263350832215	226.5			
0.000390625	1.68170355241e-005	443.7			

In both the methods the error keeps reducing with the step size. [9]

Algorithm For Runge Kutta Method of Order 4



Our project was to find a means of studying migration and its causes, in the backdrop of the migrant crises affecting the west and the middle-east currently. However, the general perception of migrants was too general to use in our analysis. We have taken up the special case of internal migration within the domain of the Republic of India and the subset of migrants who migrate in order to study in better institutions than what was available to them from their place of origin or under a scholarship/ free ship [1]. Keeping in mind the fact that many students not native to the region that they go to study in, we have attempted to make a statistical model for the implementationof an android application to allow such individuals to socialise, speak and receive help from volunteering locals. Taking into account the fact that many of the target demographic may have had troubled pasts and may be comfortable with certain types of individuals, we have attempted to not only map people to such suitable individuals, but to also over time expose these people to people outside of their "comfort zone", assuming this comfort zone to be ethnicity based [1]. We have also devised a way to take up data for abuse of the platform, and also information on volunteers who are not helpful, and then incorporate this into our mapping model to make this mapping more useful. Using a stochastic model, we will try to create a dynamic model, which may be updated based on a review system on board the application.

Result

Consider an Ordinary Differential Equation

$$\frac{dx}{dt} = 4t + 4$$

If $x=x_0$ at $t=0$ the increment in 'x' calculated using 4^{TH} order RK method with the step size of $\Delta t=0.2$

$X(t_1) \approx x_0+(k_1+2k_2+2k_3+k_4)/6$

$k_1=f(t_0, y_0)$

$k_2=f(t_0+h/2, k_1/2 + x_0)$

$k_3=f(t_0+h/2, x_0+k_2/2)$

$k_4= f(t_0+h, x_0+k_3)$

$(t_0, x_0) = (0, x_0)$

$(t_0+h/2, x_0+k_1/2) = x_1 \approx x_0+(k_1+2k_2+2k_3+k_4)/6$

$k_1=0.2*4=0.8$

$k_2=0.2[(0+0.1)*4+4]=0.88$

$k_3=0.2[(0+0.1)*4+4]=0.88$

$k_4=0.2[0.2*4+4]=0.96$

$x_1-x_0 \approx [0.8+2*0.88+2*0.88+0.96]/6=0.88$

Consider an Ordinary Differential Equation

$$\frac{dx}{dt} = 3t^2 + 2t + 9$$

If $x=x_0$ at $t=0$ the increment in 'x' calculated using 4^{TH} order RK method with the step size of $\Delta t=0.5$

$X(t_1) \approx x_0+(k_1+2k_2+2k_3+k_4)/6$

$k_1=f(t_0, y_0)$

$k_2=f(t_0+h/2, k_1/2 + x_0)$

$k_3=f(t_0+h/2, x_0+k_2/2)$

$k_4= f(t_0+h, x_0+k_3)$

$(t_0, x_0) = (0, x_0)$

$(t_0+h/2, x_0+k_1/2) = x_1 \approx x_0+(k_1+2k_2+2k_3+k_4)/6$

$k_1=0.5*4=2$

$k_2=0.5[(0+0.1)*4+4]=2.5$

$k_3=0.5[(0+0.1)*4+4]=2.5$

$k_4=0.5[0.2*4+4]=3$

$x_1-x_0 \approx [2+2.5*2+2*2.5+3]/6=2.5$

Conclusion of Results

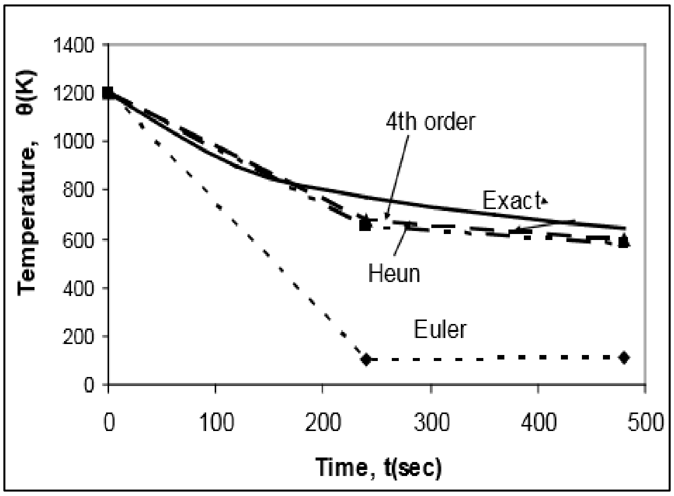
By the results obtained above we can come to a conclusion that in almost every considered differential equation the values of k_2 and k_3 is observed to be almost equal/same. Which is even the final answer obtained after solving the equation completely.

i.e. $k_2=k_3==x-x_0$.

On Comparison of Runge Kutta Method with Euler and Heun Method

we can say that Runge Kutta Method is efficient and less time consuming while Euler and Heun Methods give the approximate value in 10 and 40 steps respectively. [1]

The graph comparison is shown by



Acoustic Wave Propogation

Definition

Acoustics- concerned with properties of sound.

Or

Study of all mechanical waves in gases, liquids and solids which include vibrations, sound, ultrasound and infrasound.

Computational Aero Acoustics is focused on direct computation of aerodynamic noise.

Namely → unsteady turbulent field → radiated noise by the flow.

Acoustics is generally known as linear problem governed by the compressible linearized Euler Equations of 2,3 order to describe mean flow effects on sound propagation such as refraction.

Acoustic Waves

These are type of longitudinal waves that propagate by means of adiabatic compression and decompression. These exhibit phenomena like diffraction, reflection and interference. The propagation sound of acoustic waves is given by the speed of sound. In general, the speed of sound c is given by Newton-Laplace Equation.

Speed of sound increases with stiffness of the material and decreases with density.

Computational Aeroacoustics

Computational Aeroacoustics is a branch of aeroacoustics that aims to analyze the generation of noise by turbulent flows through numerical methods.

We have 2 types of approach for computational aeroacoustics

1)Direct Numerical Simulation Approach to Computational Aeroacoustics.

2)Hybrid Approach for Computational Fluid Dynamics.

Acoustic Wave Equation

$$\frac{Ld^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=\gamma(t)\leftrightarrow m\frac{d^2\omega}{dt^2}+r_m\frac{d\omega}{dt}+K_m\omega=f(t)$$

Electrical	Mechanical
q-charge	ω-displacement
I=dq/dt-current	V=dw/dt-particle velocity
V-voltage	F-force
L-Inductance	M-mass
R-Resistance	r_m-damping
1/c-Capacitance	K_m-Stiffness

Wave Equations

Acoustics is usually a linear problem governed by the compressible linearized Euler equations of 2,3 order to describe mean flow effects on sound propagation.

The linearized Euler equations can be written as

$$\delta P'+\mu_o\nabla P'+\mu'\nabla p_o+p_o\nabla\mu'$$

$$\rho_o(\delta_t\mu'+\mu_o\nabla\mu'+\mu'\nabla\mu_o)+\nabla P'-\left(\frac{\rho'}{\rho_o}\right)\nabla P_o=0$$

$$\delta_t S'+\mu_o\nabla S'+\mu'\nabla S_o$$

ρ – density, u- velocity, p-pressure, s-entropy

subscript o is the mean flow variables.

For a perfect gas, the linearization of the equation of the state can take the form

$p'=C_o^2\rho'+(\rho_o/C_v)\,S'$

C_v-Specific heat at constant volume

C-speed of sound

Taking a general case as for acoustics underwater, the system is closed by the relation.

$$\delta_t P'+\mu_0\nabla P'+\mu'\nabla P_o =C_o^2(\delta_t P'+\mu_0\nabla P'+\mu'\nabla P_0)+(C^2)'\mu_o\rho_0$$

When medium at rest(u_o=0), the term delρ_0 is simply the hydrostatic pressure in equilibrium with the force of gravity and is generally neglected for frequencies >> 10^{-3}Hz.

We could see some efforts to obtain approached solutions by using geometrical acoustics or parabolic approximations. [5]

The 1-dimensional advection equation can be thus considered as an archetype for numerical studies.

The equation to be solved is

$\partial_t u+C\partial_x u=0$

C-constant

The initial perturbation u(x,0) =g(x) simply propagates at the speed of C.

Solution is given by u(x-t)=g(x-ct)

Music Layers

Texture is how the tempo, melodic, and harmonic materials are combined in a composition, thus determining the overall quality of sound in piece.

Music theory has no proper foundation in modern mathematics, but the basis of musical sound can be described mathematically and exhibits “a remarkable array of number properties”

We have few elements of music like form, rhythm, meter, pitches, notes, tempo, pulse, which we can relate to measurement of time and frequency, which offers ready materials in geometry. [4]

Having tried to structure and communicate in different and unique ways of composing and hearing music it has reached to level of musical applications like set theory, abstract algebra and number theory.

Some composers have incorporated the golden ratio and Fibonacci numbers for their work.

Set Theory

Musical set theory uses the language of mathematical set theory in an elementary way to organize musical objects and describe their relationships.

To analyze this, they begin with the set of tones which forms motives and chords.

To discover deep structure in music we can apply simple operations such as transportation and inversion.

These operations are called isometries because they preserve the intervals between tones in a set.

Abstract Algebra

Rather than set theory few have used abstract algebra to analyze music. Transformation theory is a branch of music theory developed by David Lewin. It emphasizes transformations between musical objects.Theorists have given musical applications of more sophisticated algebraic concepts.

The theory of regular temperaments has been extensively developed with a wide range of sophisticated mathematics. The chromatic scale has a free and transitive action of the cyclic group Z/12Z, with the action being defined via transposition of notes. To study equal divisions of octave, theorists have used Riemann Zeta function.

Beats, Antinodes and Standing Waves

Beats is an interference pattern between 2 sounds of slightly different frequencies, perceived as a periodic variation in volume whose rate is difference of 2 frequencies. Antinodes are those points that undergo the maximum displacement during each vibrational cycle of the standing wave. These points are opposite of nodes, so they are called antinodes. Standing waves is a vibration of a system in which some points remain fixed while others between them vibrate with the maximum amplitude. These are also called stationary waves.

Mathematics and Music

Mathematics can be used to analyze musical rhythms, to analyze the sound waves that produce musical notes, to explain why instruments are tuned and to compose music.
We find relationship between mathematics and music through proportions, patterns, Fibonacci numbers or the golden ratio, geometric transformations, trigonometric functions, fractals and other mathematical concepts. [7]

Conclusion

Runge Kutta Method

- Parameters like maximum running times, step size etc. can be controlled.
- We can compare this method with other methods and even contrasts can be found (Euler and Heun Method).
- Working of numerical methods will be much clearer for students doing research on it.
- A lot more work is needed on predicting the stability and accuracy of methods for integrating nonlinear and chaotic systems.
- We compare the exact solutions and the numerical solutions, the results obtained by Runge Kutta Method fourth order are of order 10^{-20} for ϕ and 10^{-14} for θ.
- The error in numerical solution can be reduced by reducing the step size.

Acoustic Wave Propagation

- Usually, the study is regarding the development of numerical solution techniques for simulating the interaction between acoustic and elastic waves.
- Much focused on mutual interaction between 2-time harmonic linear wave equations.
- The temporal discretization for time dependent equations was made by the second-order accurate central finite different scheme or the fourth order Runge Kutta approach.

Differential Equations

- Plays a very important role in science and engineering.
- We can see its applications in our surroundings like electromagnetic theory, signal processing, computational fluid dynamics etc.
- We can solve these equations either by using analytical or numerical methods.
- Analytical solution exists rarely because it’s quite complicated to solve.

Acknowledgement

The authors thank the Department of Mathematics, St. Joseph’s College (Autonomous), Shanthinagar, Bengaluru and the faculty of the Indian Social Institute, Benson Town, Bengaluru for their support and encouragement. Special thanks to Rev. Fr. O.P. Martin and Rev. Fr. J. Joy of ISIB, for their valuable insights into the issue of migration.

Conflict of Interest

No known conflict of interest.

Funding Source

No funding was availed for this study.

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