

# Mathematical Model In Psychology - Reanalysis Of Parameters Using Logistics

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## Abstract

The paper deals with Fuzzy Logics and an implementation of Fuzzy Logics in Psychology. This was achieved by understanding the concept of Fuzzy Logics in detail and utilizing appropriate concepts of it in Psychology. It involves real life case studies of psychological depression cases and implementation of the fuzzy model on it in order to diagnose the relevant treatment required for the patient. The patient has been analysed with factors like behaviour, family, personal and social life, character, speech, thoughts, mood, personality etc after which the fuzzy models were applied on. The Fuzzy If-Then rules and Logic Gates have been found to be of great use for the diagnosis.

**Keywords:** Fuzzy logic, membership functions, if-then rules, logic gates, depression, Electroconvulsive Therapy.

## Introduction

Fuzzy Logic is a form of many valued logics in which the truth values of variables may be any real number from 0 to 1. It is employed to handle partial values like Partially True or Probably False unlike Boolean Algebra which only deals with crisp values like True or False. The approach of Fuzzy Logic imitates the way of decision making in humans that involves all intermediate possibilities between digital values YES and NO.

Fuzzy Logic was introduced by Lotfi Zadeh in the year 1965 with his proposal of Fuzzy Set Theory at the University of California. To understand Fuzzy Logic, it is important to understand Fuzzy Set Theory Mathematically.

## Fuzzy Set Theory

Fuzzy sets are simply an extension of Classical Set Theory. It contains elements that have varying degrees of membership in the set unlike having only crisp values.

Let  $A$  be a fuzzy set in the universe of information  $U$ , Now  $A$  can be defined as:

$$A = \{(y, \mu_A(y)) | y \in U\}$$

Where,  $\mu_A(y)$  = degree of membership of  $y$  in  $\{A\}$ , it assumes values in the range from 0 to 1, i.e.,  $\mu_A(y) \in [0,1]$ .

There can be two cases of universe of information and understand how a fuzzy set can be represented.

### Case 1

When universe of information  $U$  is discrete and finite

$$A = \{\mu_A(y_1)y_1 + \mu_A(y_2)y_2 + \mu_A(y_3)y_3 + \dots\}$$

$$A = \{\sum_{ni=1} \mu_A(y_i)y_i\}$$

In the above representation, the summation symbol represents the collection of each element.

### Case 2

When universe of information  $U$  is continuous and infinite –

$$A = \{\mu_A(y)y\}$$

## Membership Functions

The fuzziness of fuzzy logics is characterised by membership functions. It represents the degree of truth in fuzzy logics. Shown in Figure.1.

## Mathematical Representation:

We know that a fuzzy set  $A$  under the universe of information  $U$  can be defined as:

$$A = \{(y, \mu_A(y)) | y \in U\}$$

Here  $\mu_A(\cdot)$  = membership function of  $A$ ; this assumes values in the range from 0 to 1, i.e.,  $\mu_A(\cdot) \in [0,1]$ . The membership function  $\mu_A(\cdot)$  maps  $U$  to the membership space  $M$ .

The dot ( $\cdot$ ) in the membership function described above represents if the element in a fuzzy set is discrete or continuous.

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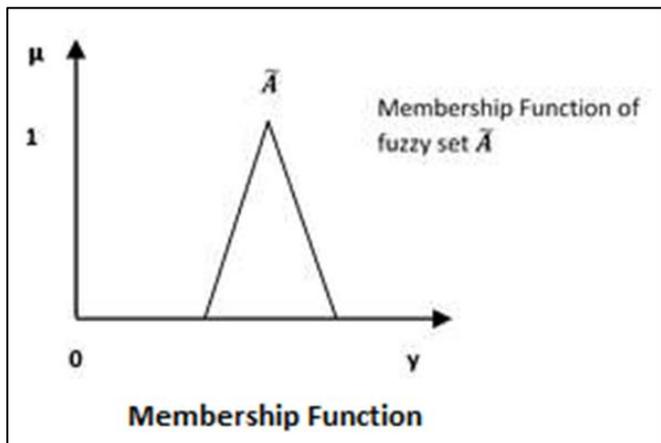
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**Interpretations Of Fuzzy If-Then Rules**

Fuzzy IF-THEN Rules can be interpreted in the following four forms

**1. Assignment Statements**

Assignment statements use “=” (equal to sign) for the purpose of assignment. They are of the following form:

$$a = \text{rainbow}$$

**2. Conditional Statements**

Conditional statements use the IF-THEN rule base form for the purpose of condition. They are of the following form:

**IF temperature is high THEN Climate is hot**

**3. Unconditional Statements**

They are of the following form:

**GOTO 75**

**Put off the switch**

**Fuzzy Logic Gates**

Basic logic gates include NOT, OR, AND, NOR, NAND, XOR and XNOR gates. Among them NOT, OR and AND are called the fundamental gates, NAND and NOR are called universal gates, XOR and XNOR are called exclusive-OR gate and exclusive –NOR gates respectively and these are used for specific purposes only. Either 0 or 1 can be used as inputs to any of the gates. Out puts are also in terms of binary numbers 0 and 1 only. Here ‘0’ indicates that no input is present (low) and ‘1’ indicates that input is present (high). Outputs of most of the logic gates can be mapped directly to probability concepts. Probability accepts only two cases about its members i.e., either the member is present or not present. Partial representation of members is not possible in probability concepts. That is the main drawback of probability applications. Fuzzy logic attempts to clear the deficiency made by probability concepts. Fuzzy logic not only allows full representation of members but also allows their partial representation in terms of degree of representation. An attempt is made to inject these Fuzzy concepts into the existing basic logic gates and analyzes their outputs. Using Fuzzy concepts in logic gates cannot be neglected as there is a lot of scope for

inputs to represent partially. All basic gates are here again re-examined with respect to Fuzzy logic concepts. Output of every basic logic gate is compared with that of the logic gate with Fuzzy concept. Inputs may be represented either by 0.1 or 0.2 or 0.26 or 0.4 etc.

**1. Boolean NOT gate:**



A is input and Q is output. Q is compliment of A. A can take 2 input values i.e. either 0 (low) or 1(high).

Input voltage	Output voltage
0	1
1	0

**Fuzzy NOT gate:**

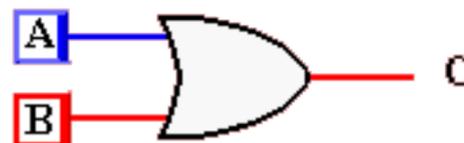
Any number of input variants can be considered.

If A is 0.8 then output Q is  $1 - 0.8 = 0.2$ .

If A is 0.5 then output Q is  $1 - 0.5 = 0.5$  etc.

Thus there are finitely many outputs exist corresponding to their inputs but not limited to only two outputs as in case of Boolean logic.

**2. Boolean OR gate:**



$$\text{Output } C = A + B$$

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

**Fuzzy OR gate:**

Here inputs A and B can be any values between 0 and 1. OR logic can be considered as Union operation in Fuzzy logic and higher value is the output of the operation.

For example,

If A is 0.4 and B is 0.8 then  $A \cup B$  is  $0.4 \cup 0.8 = 0.8$ ,

If A is 0.6 and B is 0.2 then  $A \cup B$  is  $0.6 \cup 0.2 = 0.6$  etc.

Thus it gives a wide range of outputs based on the corresponding input values.

### 3. Boolean AND gate:



$$\text{Output } C = A \cdot B$$

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

### Fuzzy AND gate:

AND logic is considered as Intersection operation in Fuzzy logic and least or lower value can be the output of the operation. For example,

if A is 0.4 and B is 0.8 then  $A \cap B$  is  $0.4 \cap 0.8 = 0.4$ ,

if A is 0.6 and B is 0.2 then  $A \cap B$  is  $0.6 \cap 0.2 = 0.2$  etc.

Thus, it gives a wide range of outputs based on the corresponding inputs.

### Fuzzy Logic: Developmental Analysis Psychology

Psychology is the scientific study of mind and behaviour. Human mind is highly complex and is hard to understand. Psychology is used in order to understand the human thought process, dreams, perceptions, emotions etc. A practising psychologist can understand if there is any problem with the human mind by handling psychological sessions.

Few psychological disorders include anxiety issues, sleep-wake disorders, feeding-eating disorder, depression, substance related or addictive disorder, personality disorder etc.

The process of medical diagnosis for the disorders has to undergo various stages of uncertainties, especially when the data is in the form of linguistic variables. Under such circumstances, using fuzzy logics concepts plays an important role extracting approximate information which may help in the diagnosis. There has been a fair amount of development in the field of fuzzy logic since 1965. There are plenty of theories on fuzzy methods on social sciences and psychology related topics such as on perception, memory, burnout systems etc. In this model we shall try to use Fuzzy Logic Developmental analysis in Psychology on the topic of depression.

### DEPRESSION

Depression is an illness that leads an individual to intense sadness including feeling helpless, worthless and hopeless. Being sad is normal, but intense sadness that affect living an individual's life is matter of concern. It leads to loss of interest in everything that was once enjoyed. Fortunately, depression can be treated depending on the level of illness.

Depression can happen at any age. It can be caused by a combination of biological, environmental, genetic and psychological factors. Depression can occur with many other serious diseases like diabetes, cancer, heart diseases etc in adults.

The symptoms of depression can vary according to an individual's personal situations. The common symptoms include:

- Felling sad always.
- Loss of interest or pleasure in activities that was once enjoyed
- Sleeplessness or oversleeping
- Suicidal thoughts or attempts
- Feeling worthless
- Facing difficulties in concentrating, making decisions, remembering or thinking
- Headaches, cramps o digestive problems
- Change in weight

### Treatment and Therapies

Depression can be treated even in the most intense case. Maximum percentage of depression patients respond well to treatments. Before starting the treatment, it is necessary for the health professional to conduct a diagnostic evaluation which includes an interview and physical examination so as to be sure of the cause of the illness. Based on the evaluation of symptoms, patient's medical and family history and environmental factors, a diagnosis plan is developed.

Treatments for Depression include:

#### 1. Medication

Antidepressants are medicines that are prescribed to treat depression. They release certain chemicals that control the mood and stress level of the individual. It takes 2-4 weeks for an antidepressant to work. It is usually prescribed for long terms for best results.

#### 2. Psychotherapy

Psychotherapy (also known as) talk therapy is used to treat mild depression. Sometimes psychotherapy and antidepressants are combined to treat moderate to severe depressions. This type of therapy includes Cognitive Behavioural Therapy (CBT). CBT helps to change behaviour and thinking. It can include only the individual or a group of people related to the situation. Depending on the complexity of the case, the treatment can 10-15 sessions or longer.

#### 3. Electroconvulsive Therapy (ECT)

If the patient does not respond to medications, then the Electroconvulsive Therapy is used. Patient receives an electrical stimulation under anaesthesia. It is given thrice a week for almost 2 or 3 weeks. ECT may cause certain side effects like memory loss, confusion, disorientation etc. Usually these side effects are short term. ECT is not painful and the

electrical impulse is not felt by the patient. It is one of the safe and effective method for majority of patients.

**4. Self-help and Coping**

Sometimes it is also possible to cure depression by few self activities. For example doing regular exercise can help one to create positive feeling and also improves mood. Quality sleep, healthy diet and avoiding alcohol can reduce depression symptoms.

**Model:**

**Patient case details:**

**Demographic details:**

Name:	RM
Age:	25
Sex:	Male
Educational Status:	B. Tech
Occupation:	Currently unemployed
Marital Status:	Unmarried
Religion:	Hindu
Socio-economic status:	Middle class
Place of living:	Urban
Informant:	Self
Information:	Reliable & inadequate

**Presenting problem:**

Not interested in anything, “Waiting for death to occur naturally. I don’t see any point in living this life.”  
(In the past 8 months)

**Case Intake:**

**Family History:**

RM is a single child, born to working parents. Relationship between his parents were conflictual. His father was addicted to alcohol. RM’s father used to physically abuse his mother when he was under the influence of alcohol. Most times of the day RM’s father was under the substance’s influence. Sometimes he would disappear only to be found on the road, naked, later by RM.

RM’s mother was concerned about looking after his father while RM took care of him necessities. Later RM’s father expired due to head injury by accident under substance influence. RM’s family didn’t receive any support from his paternal family. His maternal family, like his mother’s sister and cousins supported his family. His Uncle (maternal aunt’s husband) was playing the role of father. He expired due to some physical condition soon after he went to college. During his final year, RM’s mother expired due to a physical condition. Currently RM stays with his maternal aunt.

**History of presenting illness:**

- Duration: 8 months
- Onset: Sudden
- Course: Continuous (before 8 months, only during stressful situations)

Precipitating factor: Mother’s death; breakup with girlfriend.

Nature: Constant

In 2014, RM’s mother fell sick and was under treatment. Their relationship has been disturbed because, reportedly, the focus of their normal conversation would only be academics and performance which was unfavourable for RM. Her treatment period was really stressful for RM. He preferred to spend time with his girlfriend rather than visiting his mother or providing care for her. RM’s mother was looked after by her elder sister. On the day of RM’s final semester’s last examination, his mother passed away. RM went home and engaged in the necessary rituals.

Two weeks after that, he returned to his girlfriend’s place and helping her with her requirements (higher studies). Until he found a job, his financial needs were met by his girlfriend’s father. RM and his girlfriend were staying in Bangalore. She was studying and he was working. Unfortunately, he was fired from his job. He started isolating himself until he got the other job. The relationship was critical in this period. After few months, they started to live together. After 6-7 months, the girlfriend called off the relationship (2016). He lost hope in the purpose of living there after, reportedly.

Eight months from then, they decided to be friends with each other. But when the girl found another boyfriend, he started to feel insecure and they decided to not keep in touch and separate. He quit his job and returned to his hometown and staying with his relatives, and has been unemployed. In the recent past, he has not been engaging in any activities and there has been a difficulty in having any interest towards anything.

**Associated disturbances:**

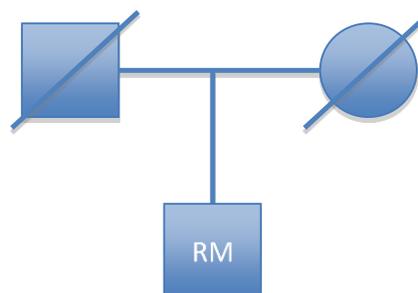
- Sleep: Premorbid
- Appetite: Increased
- Weight: Increased
- Social functioning: Impaired
- Occupational functioning: Unknown
- Sexual functioning: Premorbid

**Past psychiatric history:** Nil

**Past medical history:**

During his adolescence, he was given treatment for high blood pressure.

**Genogram:**



**Educational history:**

Reportedly, RM was an outstanding student for his involvement and participation in class. Also, for his knowledge and curiosity. Until school, his scores were also very high. In college, his academic performance reduced. Absenteeism has been constant both throughout his school and college life.

**Occupational history:**

RM switched to four different companies in a span of 2 years. He was fired from the first organization for disobeying a rule before he learnt the rules of the organization. He felt it was unjust and also blamed himself for losing his job. He joined a company after 3 months in another city (Chennai) and was irregular for his work, because he was falling sick very often due to the climatic conditions, reportedly. He resigned the job after 4 months and returned to Bangalore and joined another company. After his relationship break-up, he resigned again and joined another company which again he resigned before he left the city.

Sexual history: Premorbid  
 Substance history: Onset since break up (2016)

**Premorbid personality:**

Attitude to self: Self-pity, ego-centric, dramatizing  
 Moral & religious attitudes: Non-compliant  
 Social relationships: Adjustable but inconsistent  
 Mood: Unstable  
 Leisure time spent: Alone

**Mental status examination:**

**General appearance and behaviour:**

Appearance: Looks one's age  
 Grooming: Normal  
 Cleanliness: Adequate  
 Mode of entry: Came willingly  
 Cooperativeness: Yes

**Speech:**

Reaction time: Normal  
 Tempo: Normal  
 Volume: Normal  
 Tone: Normal  
 Coherence: Coherent  
 Relevance: Relevant

**Thought:**

Form & stream of thought: Normal  
 Possession of thought: Obsession (Present)  
 Thought content:  
 Ideas:  
 Delusion: Nil

**Mood:**

Quality: dysthymic  
 Range & reactivity: Reduced  
 Affect:  
 Subjectively: "going on"  
 Objectively: sad and disappointed

**Perception:** Nil

**Cognitive functions:**

Attention: Normal  
 Orientation: Normal to time/place/person  
 Concentration: Normal  
 Memory: Intact  
 Intelligence: Average  
 Abstraction: Adequate  
 Judgement: Adequate  
 Insight: Grade III

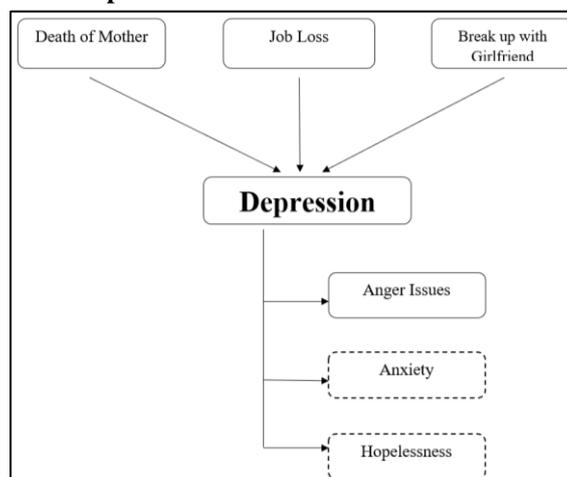
**Provisional diagnosis:**

F33.2 Major Depressive disorder, recurrent, severe without psychotic features.

**Prognosis indicator:**

Positive - Support system, Response to therapy  
 Negative – Grade III insight, obsessions, personality traits

**Flowchart Representation**



**Fuzzy Logic Developmental Analysis (IF-THEN Rules)**

Using the Fuzzy Logic IF-THEN rules, the kind of treatment required for a depression patient according to the linguistic variables (parameters) can be determined.

Linguistic variables according to the Case Study:

- Suicidal Thoughts
- Disinterested
- Happy and Carefree

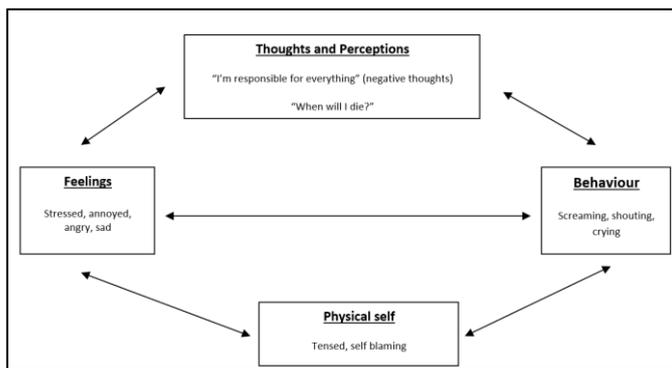
Fuzzy IF-THEN rules:

1. IF patient has suicidal thoughts, THEN the treatment is medication
2. IF patient is disinterested THEN the treatment is psychotherapy
3. IF patient is only sad THEN diagnostic evaluation is required

Using this IF-THEN Rule base of Fuzzy Logic, the case study can be analysed.

Rule 2 best suits the case. The patient is neither suicidal nor just sad. Wherein, the patient is disinterested in life and claims to wait for death to happen naturally. Such situations are best treated with Psychotherapy treatment.

Flowchart review of the case study:



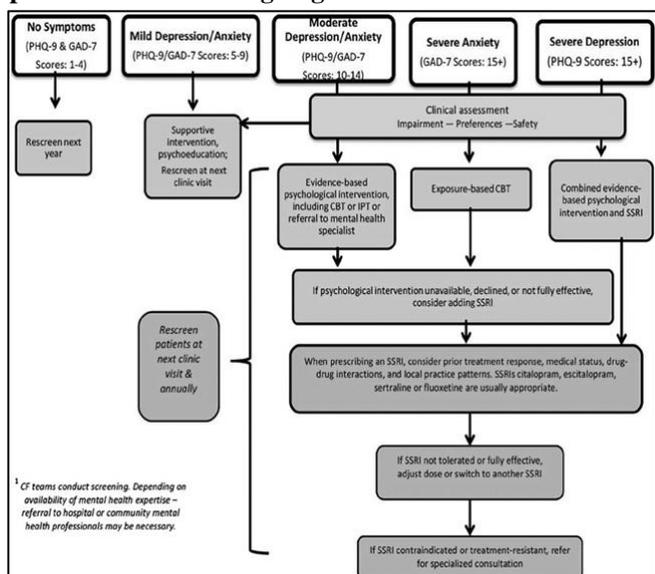
The patient tends to lose self-confidence and motivation. The thoughts are often negative which leads to behavioural issues like getting angry soon, feeling guilty, anxious, sad etc.

Hence a psychotherapy treatment plan is designed. The treatment plan includes various sessions in which the patient is counselled to face life situations. The patient's family members are also counselled for the betterment of the patient.

The treatment plan includes:

S. No.	Therapy	No. of Sessions
1	Psycho educate the patient	2
2	Psycho educate the family	2
3	Thematic apperception test (TAT) assessment	2
5	Family Therapy	2
6	Acceptance & Commitment Therapy (ACT)	3
7	Relaxation techniques	2
	Total no. of sessions	13

Optimal Model for Designing Treatment Plan



ANALYSIS OF OPTIMAL MODEL USING LOGIC GATES

Linguistic variable: Depression

Membership Functions:

Parameter 1: Based on symptoms

- No Symptoms
- Mild Depression or Anxiety
- Moderate Depression or Anxiety
- Severe Anxiety
- Severe Depression

Parameter 2: Based on treatment

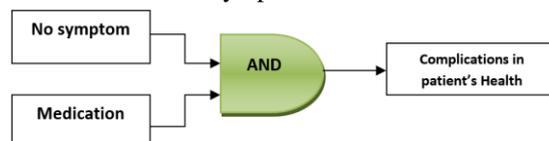
- Medication
- Psychotherapy
- Electroconvulsive Therapy

Using AND logic gate for the analysis of optimum diagnosis:

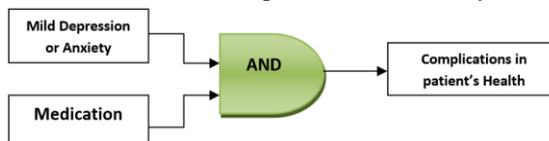
The parameter 1 and parameter 2 are sent as input to AND gate and the output is analysed if the input treatment is the optimal treatment for the input patient symptom.

1. When Parameter – 2 → Medication (fixed) and Parameter – 1 varies

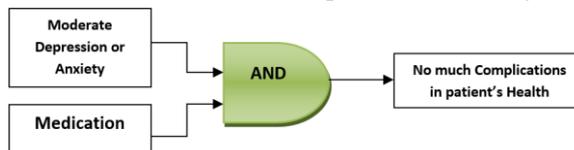
i. Parameter – 1 → No symptom



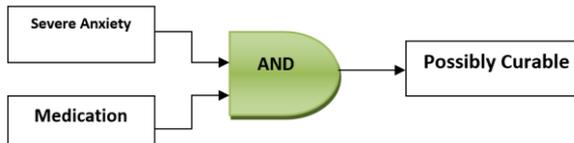
ii. Parameter – 1 → Mild Depression and Anxiety



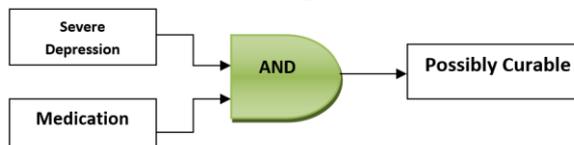
iii. Parameter – 1 → Moderate Depression or Anxiety



iv. Parameter – 1 → Severe Anxiety



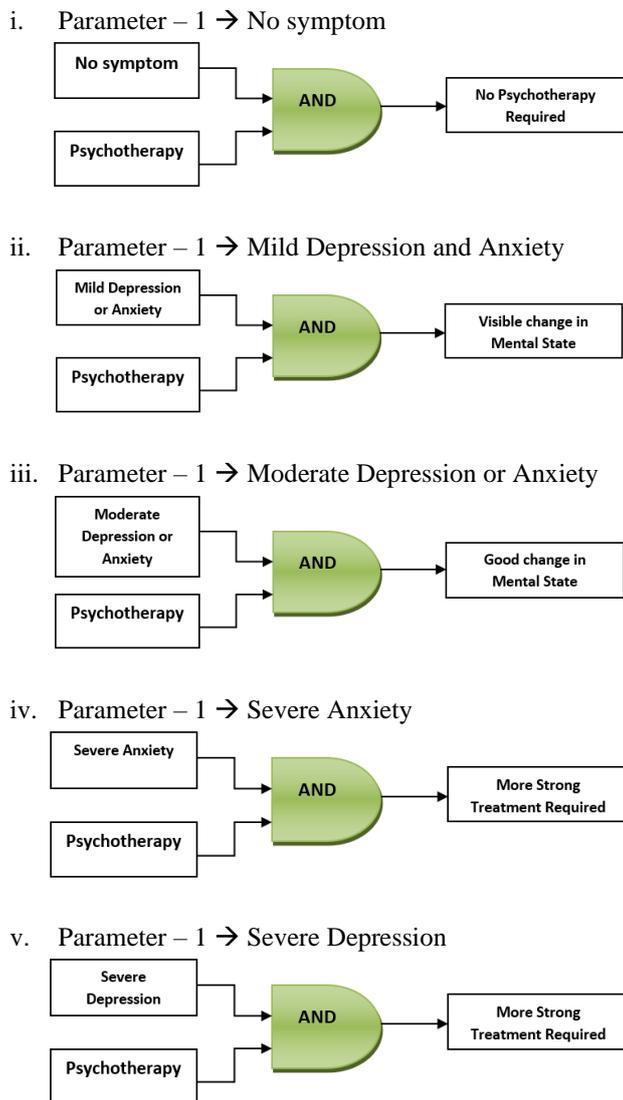
v. Parameter – 1 → Severe Depression



Hence it is clearly evident that Medication is the treatment only for Severe Anxiety or Depression cases. Some extreme

cases may not only require medication but also might need Electroconvulsive Therapy for better treatment result.

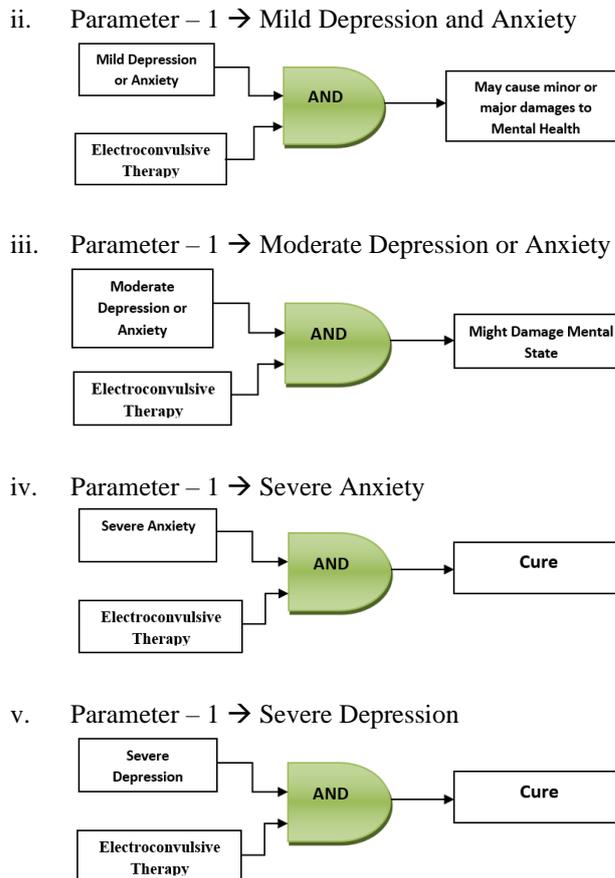
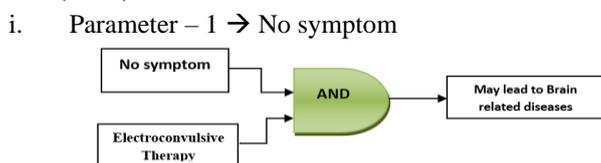
**2. When Parameter – 2 → Psychotherapy (fixed) and Parameter – 1 varies**



It is evident from these gates that Psychotherapy is the optimal treatment only for Mild and Moderate level of Depression or Anxiety. When the level of the illness is more psychotherapy is not enough. Additionally, medication or electroconvulsive therapy is required to diagnose high level cases.

It is evident from these gates that Psychotherapy is the optimal treatment only for Mild and Moderate level of Depression or Anxiety. When the level of the illness is more psychotherapy is not enough. Additionally, medication or electroconvulsive therapy is required to diagnose high level cases.

**3. When Parameter – 2 → Electroconvulsive Therapy (fixed) and Parameter – 1 varies**



Electroconvulsive Therapy (ECT) is a treatment given for patient with higher level of Depression or Anxiety. It is also evident from the logic gate that ECT is an optimal solution for severe depression. It is also understood that it is dangerous to recommend ECT for mild level of the illness.

**Conclusion**

**Human Behaviour and Psychology**

Each individual has a different way of reacting to different situations. It depends on social norms, attitude, genetics, core faith etc. In psychology, it is important to understand the person and their background before treating them. The person’s beliefs, values, way of thinking, lifestyle are the parameter to be looked upon for understanding human behaviours.

All small actions of an individual create an impact on understanding their behaviour. It requires keen observation and analysis before coming to a conclusion. It is also possible for a psychotherapist to go wrong in their observations. This is taken care by fuzzy models. Fuzzy models contain certain rules and restrictions hence ensuring optimal treatment measures. Using fuzzy logic models provide more reliable outputs.

**Impact of Fuzzy Logics on Psychology**

Analysis of psychological disorders using fuzzy logics helps in better diagnosis. It is important to treat the patient with the best treatment possible. The possible parameters are analysed using Fuzzy Logic Rules and Fuzzy Logic Gates.

The treatment plan for the case study considered in the model, has given an appreciable improvement in the patient's mental state. It is now observed that the patient is optimistic and is coming out of depression gradually. Hence it is evident that fuzzy models are of great use in diagnosing patients. It can not only be used in psychology but also in various medical fields.

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### Conflict of Interest

No known conflict of interest.

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